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FINAL REVIEW - TRUE, FALSE, OR NONSENSE?

For each statement below, decide as a group whether it is true, false, or nonsense (i.e. its truth cannot be evaluated, because some or all of the terms are used incorrectly). Write T, F, or N on the line. If a statement is true, explain briefly. If it is false or nonsense, make a slight adjustment to it so that it makes sense and is true. Turn in one copy per group.

2 ✓(1) T The following system of linear equations is inconsistent:

$$\left\{ \begin{array}{l} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 4 \\ 3x_2 - 3x_3 = 3 \end{array} \right. \quad \left| \begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 3 & -3 & 3 \end{array} \right. \sim \left| \begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & -1 & 1 \end{array} \right. \sim \left| \begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 0 & 3 \end{array} \right. \quad \text{inconsistency of 3}$$

2 ✓(2) N Given a subspace W of \mathbb{R}^n , W^\perp spans the same basis as W .
bases span subspaces!

2 ✓(3) T The basis that spans w does not span w^\perp
If A is an $n \times n$ matrix such that A^T is invertible, then the product of the eigenvalues of A is nonzero. $\det A = \det A^T$ invertible if $\det A = \det A^T \neq 0$
by IMT, A invertible if $\lambda \neq 0$. So $\lambda=0$ cannot be eigenvalue, so their product $\neq 0$.

2 ✓(4) F Given three functions $a(t)$, $b(t)$, and $c(t)$, continuous and defined on $(-\infty, \infty)$, the differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$.

To get exactly one solution for an n th order diff. eq., you need $n-1$ initial conditions. We need a $y(t_0) = b_0$ to get a unique solution.

2 ✗(5) F If $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a basis for V , then $A = (\bar{v}_1 \ \dots \ \bar{v}_n)$ must be a square matrix.

$V = \mathbb{R}^n$
good fly

2 ✓(6) N For any three vectors \bar{u} , \bar{v} , and \bar{w} in \mathbb{R}^n , $(\bar{u} \cdot \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \cdot \bar{w})$.

$$(\bar{u} \cdot \bar{v}) \bar{w} \neq (\bar{v} \cdot \bar{w}) \bar{u}$$

↑
scalar mult.

2 ✓(7) T If A is an $n \times n$ matrix, and $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a set of eigenvectors with distinct eigenvalues, then A is diagonalizable.

Theorem 7 in ch. 6

2 ✓(8) F If A and B are $n \times n$ matrices, $|A+B| = |A| + |B|$.

$$|AB| = |A||B|$$

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FINAL REVIEW - TRUE, FALSE, OR Nonsense?

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2

(1) T The following system of linear equations is inconsistent:

*Solutions of Inconsistent system
returning since there are only
2 points.*

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 4 \\ 3x_2 - 3x_3 = 3 \end{cases} \quad \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 2 & -3 & 3 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -2 & 2 & 4 \\ 0 & 2 & -3 & 3 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & 1 & 1 \end{array} \right)$$

0

N F Given a subspace W of \mathbb{R}^n , W^\perp spans the same basis as W .

*would not necessarily span the same basis as W . since W may not
span W^\perp even though $W \oplus W^\perp$ are both subspaces of \mathbb{R}^n*

2

(3) T If A is an $n \times n$ matrix such that A^T is invertible, then the product of the eigenvalues of A is nonzero. *if A^T is invertible $\det |A^T| \neq 0$, the product of eigenvalues determine the determinant so eigenvalues' products $\neq 0$*

2

(4) F Given three functions $a(t)$, $b(t)$, and $c(t)$, continuous and defined on $(-\infty, \infty)$, the differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$.

True: if there is given an initial value

2

(5) N If $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a basis for V , then $A = (\bar{v}_1 \dots \bar{v}_n)$ must be a square matrix.

*(?) V in what subspace? If V is in \mathbb{R}^n , then this would
be true.*

2

(6) N For any three vectors \bar{u} , \bar{v} , and \bar{w} in \mathbb{R}^n , $(\bar{u} \cdot \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \cdot \bar{w})$.

*cannot do dot product with a scalar and a vector
instead, $(\bar{u} \cdot \bar{v})\bar{w} = \bar{u} \cdot (\bar{v}\bar{w})$*

2

(7) T If A is an $n \times n$ matrix, and $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a set of eigenvectors with distinct eigenvalues, then A is diagonalizable.

definitable of diagonalization

2

(8) F If A and B are $n \times n$ matrices, $|A + B| = |A| + |B|$.

$|AB| = |A||B|$. There is nothing good to say about $|A+B|$

Adam Midvidy

James Chrzancky

13/16

Group Number: 4

Franzua

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2 ✓ T

The following system of linear equations is inconsistent:

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 4 \\ 3x_2 - 3x_3 = 3 \end{cases}$$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

ON X F

Given a subspace W of \mathbb{R}^n , W^\perp spans the same basis as W .
subspace Row(A) of $\text{Row}_1(A)$ spans the same basis as $\text{Nul}(A)$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -2 & 2 & 4 \\ 0 & 3 & -3 & 3 \end{bmatrix}$$

Inconsistent

2 ✓ T

If A is an $n \times n$ matrix such that A^T is invertible, then the product of the eigenvalues of A is nonzero.

then no eigenvalue is 0, so the matrix is invertible

2 ✓ F

Given three functions $a(t)$, $b(t)$, and $c(t)$, continuous and defined on $(-\infty, \infty)$, the differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$.

only if i.e. value specified
 $y(b) \in (-\infty, \infty)$

1 ✓ N F

If $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a basis for \mathbb{R}^n , then $A = (\bar{v}_1 \dots \bar{v}_n)$ must be a square matrix.

not a matrix if $\bar{v}_1, \dots, \bar{v}_n$ not in \mathbb{R}^n

$(V+V) \cdot W = V \cdot (V+W)$ what is it?

$$(\bar{U}+\bar{V})+\bar{W} = \bar{U}+(\bar{V}+\bar{W})$$

2 ✓ N

For any three vectors \bar{u} , \bar{v} , and \bar{w} in \mathbb{R}^n , $(\bar{u} \cdot \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \cdot \bar{w})$.

$$(U+V) \cdot W = U \cdot (V+W)$$

2 ✓ T

If A is an $n \times n$ matrix, and $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a set of eigenvectors with distinct eigenvalues, then A is diagonalizable.

by definition

the eigenvectors form P

and D is $\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_n \end{bmatrix}$

$$|A+B|$$

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- one copy per group.

(1) The following system of linear equations is inconsistent:

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 3 & -3 & 3 \end{array} \right]$$

see here - $\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 4 \\ 3x_2 - 3x_3 = 3 \end{cases}$

(2) Given a subspace W of \mathbb{R}^n , W^\perp spans the same basis as W .

(3) If A is an $n \times n$ matrix such that A^T is invertible, then the product of the eigenvalues of A is nonzero.

If it is zero, one of the e-values is 0. Then, it has a nullspace of at least one vector. So A^T cannot be invertible. This is true, we don't know this.

(4) Given three functions $a(t)$, $b(t)$, and $c(t)$, continuous and defined on $(-\infty, \infty)$, the differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$.
a general solution

(5) If $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a basis for \mathbb{R}^n , then $A = (\bar{v}_1 \dots \bar{v}_n)$ must be a square matrix.

For any three vectors \bar{u} , \bar{v} , and \bar{w} in \mathbb{R}^n , $(\bar{u} \cdot \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \cdot \bar{w})$.
commutative property makes this true.

(6) If A is an $n \times n$ matrix, and $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a set of eigenvectors with distinct eigenvalues, then A is diagonalizable.
I'll give it to you. It means it's diagonalizable if the algebraic multiplicities of its eigenvalues add up to n .

(7) If A and B are $n \times n$ matrices, $|A + B| = |A| + |B|$.
 $|AB| = |A||B|$

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Group Number: 1

FINAL REVIEW - TRUE, FALSE, OR NONSENSE?

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2 T The following system of linear equations is inconsistent:

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 4 \\ 3x_2 - 3x_3 = 3 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 3 & -3 & 3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -2 & 2 & 4 \\ 0 & 3 & -3 & 3 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 2 \\ 0 & 1 & -1 & 1 \end{array} \right) = \boxed{\text{No solution}}$$

ON F Given a subspace W of \mathbb{R}^n , W^\perp spans the same basis as W .

Nul(W^T) This doesn't mean anything!

1 T If A is an $n \times n$ matrix such that A^T is invertible, then the product of the eigenvalues of A is nonzero.

If A^T has inverse, A has inverse $\Rightarrow A$ diagonal w/e values on diagonal and initial value information $y(0)=a$ but if it's differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$. Then A not inv.

2 F Given three functions $a(t)$, $b(t)$, and $c(t)$, continuous and defined on $(-\infty, \infty)$, the differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$. Then A not inv.

ON F If $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a ^{n-dimensional} basis for V , then $A = (\bar{v}_1 \ \dots \ \bar{v}_n)$ must be a square matrix.

2 N For any three vectors \bar{u} , \bar{v} , and \bar{w} in \mathbb{R}^n , $(\bar{u} + \bar{v}) + \bar{w} = \bar{u} + (\bar{v} + \bar{w})$.

$$(\bar{u} + \bar{v}) + \bar{w} = \bar{u} + \bar{v} + \bar{w} = \bar{u} + (\bar{v} + \bar{w})$$

2 T If A is an $n \times n$ matrix, and $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a set of eigenvectors with distinct eigenvalues, then A is diagonalizable.

if n distinct values \Rightarrow n distinct eigenvectors \Rightarrow linearly independent, thus $A = PDP^{-1}$

2 F If A and B are $n \times n$ matrices, $|A + B| = |A| + |B|$. P matrix of e-vectors $|AB| = |A||B|$ D matrix of e-values

Jun Jin
Chris
Goodman
other Chris
Doebla

10/16

Group Number: 6

FINAL REVIEW - TRUE, FALSE, OR NONSENSE?

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Q1) T The following system of linear equations is inconsistent:

$$\begin{cases} x_1 + 2x_2 - x_3 = 0 \\ 2x_1 + 2x_2 = 4 \\ 3x_2 - 3x_3 = 3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 2 & 2 & 0 & 4 \\ 0 & 3 & -3 & 3 \end{array} \right] \xrightarrow{\text{Row operations}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & -1 & 1 \end{array} \right]$$

Q2) F Given a subspace W of \mathbb{R}^n , W^\perp spans the orthogonal complement of W .

Q3) T If A is an $n \times n$ matrix such that A^T is invertible, then the product of the eigenvalues of A is nonzero.

0 is not an eigenvalue if the system is inconsistent
1. nearly independent. If A^T is invertible
so is A and is also (nearly independent)
All eigenvalues are non-zero

Q4) N Given three functions $a(t)$, $b(t)$, and $c(t)$, continuous and defined on $(-\infty, \infty)$, the differential equation $a(t)y' + b(t)y = c(t)$ has exactly one solution defined on $(-\infty, \infty)$.

at least no, always infinitely many.

Q5) E If $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a basis for \mathbb{R}^n , then $A = (\bar{v}_1 \dots \bar{v}_n)$ must be a square matrix.

Q6) N For any three vectors \bar{u} , \bar{v} , and \bar{w} in \mathbb{R}^n , $(\bar{u} \cdot \bar{v}) \cdot \bar{w} = \bar{u} \cdot (\bar{v} \cdot \bar{w})$.

Fix?

can't take the dot product
of a dot product

Q7) T If A is an $n \times n$ matrix, and $\{\bar{v}_1, \dots, \bar{v}_n\}$ is a set of eigenvectors with distinct eigenvalues, then A is diagonalizable.

A eigen vectors in an AKA matrix gives n eigenvalues
which satisfies the multiplication for diagonalization

Q8) F If A and B are $n \times n$ matrices, $|A \otimes B| = |A| \otimes |B|$.

$$A = [V_1 \dots V_n] [L_1 \ 0 \ \dots \ 0] [V_1 \dots V_n]$$